Worcester County Mathematics League

Freshman Meet 2 – February 2022
Round 1: Algebraic Word Problems

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1

1. A woman is 20 years older than her nephew. Six years from now the woman will be twice the nephew's age. How old is the woman now?

2. Megan tutors three students (Antonio, Alexy and Mikayla) for a total of 8 hours each week. She charges Alexy $4 more per week than Antonio. She charges Mikayla 1½ times the amount she charges Alexy per week. If Megan earns a total of $52 per week, how much does Mikayla pay (in dollars)?

3. A copy machine can copy a report in 30 minutes. If a second copy machine were to be used at the same time, the copying of the report would be completed in 18 minutes. How long would it take the second copy machine to complete the copying alone (in minutes)?
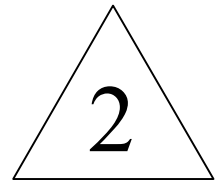
ANSWERS

(1 pt.)   1._____ years old

(2 pts.)   2. $ _____

(3 pts.)   3._____ minutes

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Round 2: Number Theory
All answers must be in simplest exact form
NO CALCULATOR ALLOWED
Note: A subscript indicates the number's base.

2

1. Complete the following base-2 addition problem and write your answer in base-10:

$$
\begin{array}{rrrrrr}
1 & 0 & 1 & 0 & 0 & 1_2 \\
1 & 1 & 0 & 0 & 1 & 1_2 \\
 & 1 & 1 & 1 & 0 & 0_2 \\
 & & 1 & 0 & 1 & 0_2 \\
+ & & & 1 & 0 & 0_2 \\
\hline
\end{array}
$$

2. Find a two-digit prime number such that the product of the number's digits is equal to seven more than the sum of the number's digits.

3. What is the largest 3-digit number which yields the same non-zero remainder when divided by 5 as when divided by 7?

ANSWERS

(1 pt.)   1.＿＿＿＿＿＿＿＿＿＿＿＿ (base-10)

(2 pts.)   2.＿＿＿＿＿＿＿＿＿＿

(3 pts.)   3.＿＿＿＿＿＿＿＿＿＿

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Round 3: Operations on Fractions, Decimals, Percents and Percentage Word Problems
All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. $66\frac{2}{3}\%$ of Q is the number that is $16\frac{2}{3}\%$ of 84. Find the value of Q.

2. How many ounces of a 30% copper alloy must be melted with a 70% copper alloy in order to produce 20 ounces of a 46% copper alloy?

3. The following addition problem involves two repeating decimals. Simplify the expression as a fraction reduced to lowest terms.

$$0.0\overline{6} + 0.0\overline{5} + \frac{5}{6}$$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____ ounces

(3 pts.)   3._____

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Round 4: Set Theory

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. For this problem, the universal set is $U = \{0,1,2,3,4,5,6,7,8,9\}$. Let E and T be subsets of U such that $E = \{$even integers$\}$ and $T = \{$multiples of $3\}$. List the elements in the set $\left(E' \cap T\right) \cup \left(E \cap T'\right)$, where S' indicates the complement of set S.

2. How many subsets can be formed from the set $\{a, b, c, d, e, f, g\}$ if no subset contains more than 2 elements?

3. Earlier this semester a random sample of 200 math majors across the nation showed that: 70% of the students took linear algebra, 47% of the students took calculus, 29% took computers, 21% took calculus and linear algebra, 19% took linear algebra and computers, 18% took calculus and computers, and 12% took all three courses. How many of the students took only computers (and not linear algebra or calculus at the same time)?

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____ subsets

(3 pts.)   3._____ students

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded correctly to at least three decimal places</u>!   (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. How many distinct triangles can be constructed by connecting three different vertices of a cube?

2. Find the sum of the largest negative and the smallest positive solutions to the inequality $\left| \dfrac{2x + 6}{5} \right| \geq 4$.

3. Anne has three times as many M&M's as Brad has. She gives him 10 M&M's, but now she has twice as many as he does. How many more M&M's must Anne give to Brad in order for the two of them to have exactly the same number of M&M's?

4. When the expression $(2x^2 - 3x + 4)^2 - (2x^2 - 5x + 4)^2$ is expanded and written as a single polynomial, what is the sum of the polynomial's coefficients?

5. Alice sells an item for $10 less than the list price and receives 10% of her selling price as a commission. Bill sells the same item for $10 less than Alice's selling price and receives 20% of his selling price as a commission. If Bill's commission is $10 more than Alice's commission, what is the list price of the item (in dollars)?

6. A date is called *weird* if the number of its month and the number of its day have a greatest common factor of 1. For example, April 15$^{\text{th}}$ is weird because the greatest common factor of 4 and 15 is 1. What month has the fewest number of weird days and how many weird days are in that month?

7. For this problem let the universal set be $U = \{1, 2, 3, 4, 5, 6\}$. Let A, B and C be sets such that $A \cup C = \{1, 2, 3, 4, 5, 6\}$, $B \cup C = \{1, 2, 3, 4\}$, $A \cap C = \varnothing$, $A \cap B = \{3\}$, and $B \cap C = \{1, 2\}$. List the elements in set B.

8. What are the coordinates of the reflection of $(6, 0)$ across the graph of the line $y = 3x$?

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ANSWERS
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Round 1
1. 34

2. 24

3. 45

Round 2
1. 134

2. 29 or 53
    (accept either 1 or both of these answers)

3. 984

Round 3
1. 21

2. 12

3. $\dfrac{43}{45}$ (only)

Round 4
1. 2, 3, 4, 8, 9   OR   {2, 3, 4, 8, 9}
    (need all 5 elements in any order)

2. 29

3. 8

Team Round

1. 56

2. −6

3. 20

4. 8

5. 130

6. June   10
    (need both answers for the 2 answer spaces)

7. 1, 2, 3   OR   {1, 2, 3}
    (need all 3 elements in any order)

8. $(-4.8, 3.6) = \left(-4\frac{4}{5}, 3\frac{3}{5}\right)$
    $= \left(-\dfrac{24}{5}, \dfrac{18}{5}\right)$